

Cosmological Electromagnetic Fields due to Gravitational Wave Perturbations

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We consider the dynamics of electromagnetic fields in an almost-Friedmann-Robertson-Walker universe using the covariant and gauge-invariant approach of Ellis and Bruni. Focusing on the situation where deviations from the background model are generated by tensor perturbations only, we demonstrate that the coupling between gravitational waves and a weak magnetic test field can generate electromagnetic waves. We show that this coupling leads to an initial pulse of electromagnetic waves whose width and amplitude is determined by the wavelengths of the magnetic field and gravitational waves. A number of implications for cosmology are discussed, in particular we calculate an upper bound of the magnitude of this effect using limits on the quadrupole anisotropy of the Cosmic Microwave Background.

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I. INTRODUCTION

There have been numerous investigations on the scattering of electromagnetic waves off gravitational fields (See Refs. [1] for a representative sample). Most of this research have been focused on the effect gravitational waves have on vacuum electromagnetic fields. Other papers, see e.g. Refs. [2], also consider situations in Astrophysics where plasma effects are taken into account.

Magnetic fields play an important role in our Universe, appearing on all scales from the solar system, through interstellar and extra-galactic scales, to intra-cluster scales of several Mpc. Although magnetic field inhomogeneities have not yet been observed on scales as large as those exhibited by Cosmic Microwave Background (CMB) anisotropies, it is natural to expect that magnetic fields exists on such scales [3], and that they could play a role in the formation of large-scale structure. Indeed many mechanisms have been proposed to explain how these fields may be generated in the early universe - a process called *primordial magnetogenesis*. For example on small scales (less than the Hubble radius), QCD and Electro-Weak transitions can give rise to local charge separation leading to local currents which can generate magnetic fields [4]. Large-scale magnetic fields can be generated during inflation or in pre-Big Bang models based on string theory [5], in which vacuum fluctuations are amplified via the inflaton or dilaton.

The effect magnetic fields have on density perturbations has been studied extensively by a number of authors, both in the context of Newtonian and Relativistic Cosmology [6,7], but as yet there have been no studies of their effect on gravitational wave perturbations.

In what follows, we use the well known covariant and gauge-invariant approach of Ellis and Bruni [8] to in-

vestigate this interaction in the context of cosmology by considering the dynamics of electromagnetic fields in an almost Friedmann-Robertson-Walker (FRW) universe, focusing in the situation where deviations from the FRW background are generated by tensor or gravitational wave perturbations [9,10].

We show that in the presence of a weak (near-homogeneous) magnetic test field propagating on the background FRW model [11], the gravitational waves couple non-linearly to this field to produce a pulse of gravitationally induced electromagnetic waves. In particular, because of the different ways in which tensor perturbations enter the wave equations for the electric and magnetic fields, respectively, there will be, in the case of long wavelength gravitational waves and large-scale magnetic fields, a growth in the expansion normalised electric field, as the expansion normalised shear grows in time.

This paper is organised as follows. After a short discussion of notation and conventions, in section III we outline in detail the linearisation procedure used to approximate the Einstein-Maxwell equations in cosmology. In section IV we derive a set of non-linear wave equations which show how electromagnetic fields can be generated when gravitational waves couple to a near-homogeneous magnetic test field propagating on the FRW background. Finally in section V we solve the equations perturbatively and use our results to put an upper bound on the size of these gravitationally induced fields.

II. NOTATION AND CONVENTIONS

Notation and conventions are taken to be the same as in [12]. In particular $8\pi G = c = 1$; the projected

spatial covariant derivative a tensor $T^{cd...}_{ef...}$ is given by $\tilde{\nabla}_a T^{cd...}_{ef...} \equiv h^b{}_a h^c{}_p \dots h^d{}_q h^r{}_e \dots h^s{}_f \nabla_b T^{p\dots q}_{r\dots s}$, where u^a is the 4-velocity of the matter, g_{ab} is the metric tensor and $h_{ab} = g_{ab} + u_a u_b$ is the spatial projection tensor ($h_{ab} u^b = 0$). A dot denotes the covariant derivative along u^a , so for any tensor $\dot{T}^{cd...}_{ef...} \equiv u^a \nabla_a T^{cd...}_{ef...}$. We assume that the matter is described by *irrotational dust* [13] so that the *pressure* p , *acceleration vector* \dot{u}^a and *vorticity tensor* ω_{ab} all vanish exactly. In this case the first covariant derivative of the 4-velocity can be written as $\nabla_a u_b = \sigma_{ab} + \frac{1}{3}\Theta h_{ab}$, where σ_{ab} and Θ are respectively the usual shear and volume expansion of the matter congruence. We also define the Hubble parameter H terms of the expansion Θ and scale factor a in the usual way: $H = \Theta/3 = \dot{a}/a$.

III. APPROXIMATIONS

In order to simplify the non-linear dynamics of the coupled Einstein-Maxwell equations and to isolate the effects we are looking for, we will adopt the following *approximation scheme* based on two parameters: ε_g will refer to quantities occurring in the gravitational equations, while ε_{em} characterise the electromagnetic field. We assume that the gravitational equations follow the *almost FRW* conditions [8], so that the energy density μ and expansion Θ have a non-zero contribution in the background model and can therefore be considered $\mathcal{O}(0_g)$ while σ_{ab} , E_{ab} , and H_{ab} vanish in the background and are $\mathcal{O}(\varepsilon_g)$. In the case of the Maxwell field, we assume that there is a *weak magnetic test field* B_0^a at $\mathcal{O}(0_{em})$ which propagates on the background FRW model, whose gravitational influence is given by the Alfvén parameter $\varepsilon \equiv (B_0^a B_{0a}^0/\mu)^{1/2}$. On the other hand the electric field E^a vanishes in the background and is considered to be $\mathcal{O}(\varepsilon_{em})$. The perturbation scheme we adopt is to drop terms of $\mathcal{O}(\varepsilon^2)$ (so that the magnetic field does not contribute to the gravitational dynamics [7]), $\mathcal{O}(\varepsilon_g^2)$, $\mathcal{O}(\varepsilon_{em}^2)$, and $\mathcal{O}(\varepsilon_g \varepsilon_{em})$.

In the covariant approach to linear perturbations of FRW models [8], *pure tensor* or *gravitational wave* perturbations are characterised by the following covariant conditions [9,10]:

$$\tilde{\nabla}^b E_{ab} = 0 \Rightarrow \tilde{\nabla}_a \mu = 0, \quad (1a)$$

$$\tilde{\nabla}^b H_{ab} = 0 \Rightarrow \omega_a = 0, \quad (1b)$$

the first one excludes scalar (density) perturbations and the second, vector (rotational) perturbations. The conditions that the terms on the right hand side vanish, are analogous to the transverse condition on tensor perturbations in the metric approach. In addition, we notice that since the Weyl tensor is the trace-free part of the Riemann tensor, both E_{ab} and H_{ab} are trace-free, again

like the tensor perturbations of the Bardeen approach [14].

Given the assumed equation of state, these conditions also imply that the spatial gradient of the expansion $\tilde{\nabla}_a \Theta$ vanish (see [8]). Together with (1) these conditions provide a unique characterisation of tensor perturbations.

IV. EINSTEIN - MAXWELL EQUATIONS

We assume overall charge neutrality and use the Bianchi identities and Maxwell's equations as presented in [12]. Then, with the above prerequisites, we obtain a set of non-linear wave equations for the gravitational (σ_{ab} : shear) and electromagnetic (E^a , B^a : electric and magnetic fields) degrees of freedom:

$$\Delta \sigma_{ab} + 5H \dot{\sigma}_{ab} + \frac{3}{2} H^2 \sigma_{ab} = 0, \quad (2a)$$

$$\Delta E^a + 5H \dot{E}^a + 3H^2 E^a + \dot{j}^a + \Theta j^a = j_E^a, \quad (2b)$$

$$\Delta B^a + 5H \dot{B}^a + 3H^2 B^a - \epsilon^{abc} \tilde{\nabla}_b j_c = j_B^a, \quad (2c)$$

where

$$\begin{aligned} j_E^a &= \epsilon^{abc} \tilde{\nabla}_b (\sigma_c^d B_d) + \epsilon^{abc} \sigma^d{}_b \tilde{\nabla}_d B_c + H^a{}_b B^b, \\ j_B^a &= 2H \sigma^a{}_b B^b - 2E^a{}_b B^b + \sigma^a{}_b \dot{B}^b \end{aligned} \quad (3)$$

are gravitational induced magnetic and electric currents, and $E_{ab} = -\dot{\sigma}_{ab} - \frac{2}{3}\Theta \sigma_{ab}$ and $H^{ab} = \epsilon^{cd(a} \tilde{\nabla}_c \sigma^{b)}$ are the electric and magnetic parts of the Weyl tensor. Also $\Delta f \equiv \ddot{f} - \tilde{\nabla}^2 f$ where f is any tensor orthogonal to u^a .

In the above equations, the electric and magnetic fields consist of two parts, a contribution due to the magnetic test field B_0^a which gives rise to the current j^a [15] and contributions generated by the non-linear coupling of this test field to gravitational waves via the gravitationally induced currents j_E^a and j_B^a :

$$E^a = E_{grav}^a, \quad B^a = B_0^a + B_{grav}^a. \quad (4)$$

V. ANALYTIC SOLUTIONS AND NUMERICAL INTEGRATION

We solve the above equations perturbatively by first calculating the gravitationally induced currents j_E^a and j_B^a and then solving (2b) and (2c) together with (2a) for the gravitationally induced electric and magnetic fields.

To $\mathcal{O}(0_{em})$, Maxwell's equations [12] give $E = 0$ and

$$\dot{B}_0^a + 2H B_0^a = 0, \quad (5)$$

which we can integrate to obtain:

$$B_0^a = a^{-2} A_{(n)}^a, \quad \tilde{\nabla}^a B_0^b = a^{-3} A_{(n)}^{ab}, \quad A_{(n)}^{ab} \equiv a \tilde{\nabla}^a A_{(n)}^b, \quad (6)$$

where $A_{(n)}^a$ and $A_{(n)}^{ab}$ determine the spatial variation of the magnetic test field and are constant along the fluid flow lines: $\dot{A}_{(n)}^a = \dot{A}_{(n)}^{ab} = 0$ [10]. Furthermore we assume that the spatial functions $A_{(n)}^a$ and $A_{(n)}^{ab}$ satisfy the Helmholtz equation

$$\tilde{\nabla}^2 A_{(n)}^a = -\frac{n^2}{a^2} A_{(n)}^a, \quad \tilde{\nabla}^2 A_{(n)}^{ab} = -\frac{n^2}{a^2} A_{(n)}^{ab}, \quad (7)$$

in this way defining a specific length scale $\lambda_{B_0} = 2\pi a/n$ associated with the magnetic field determining its scale of inhomogeneity, where n is a *fixed* wavenumber associated with that scale.

In order to solve equations (2) it is standard to decompose physical (perturbed) fields into a spatial and temporal part using eigenfunctions which are solutions of the Helmholtz equation [16]. In the case of the shear tensor we write

$$\sigma_{ab} = \sum_k \sigma_{(k)} Q_{ab}^{(k)}, \quad \dot{Q}_{ab}^{(k)} = 0, \quad (8)$$

where $Q_{ab}^{(k)}$ is a tensor harmonic satisfying

$$\tilde{\nabla}^2 Q_{ab}^{(k)} = -\frac{k^2}{a^2} Q_{ab}^{(k)}. \quad (9)$$

We can also define higher order harmonics by taking comoving spatial derivatives of the lower order harmonics, for example $Q_{abc}^{(k)} \equiv a \tilde{\nabla}_a Q_{bc}^{(k)}$ can easily be shown to satisfy

$$\tilde{\nabla}^2 Q_{abc}^{(k)} = -\frac{k^2}{a^2} Q_{abc}^{(k)}. \quad (10)$$

Using the above solution (6) (dropping the index n which indicates the scale length of B_0^a), the decomposition (8) and writing

$$E_{\text{grav}}^a = \sum_k \mathcal{E}_{(k)} \mathcal{E}_{(k)}^a, \quad B_{\text{grav}}^a = \sum_k \mathcal{H}_{(k)} \mathcal{H}_{(k)}^a, \quad (11)$$

the wave equations (2) become

$$\ddot{\sigma}_{(k)} + 5H\dot{\sigma}_{(k)} + \left(\frac{3}{2}H^2 + \frac{k^2}{a^2}\right) \sigma_{(k)} = 0, \quad (12a)$$

$$\ddot{\mathcal{E}}_{(k)} + 5H\dot{\mathcal{E}}_{(k)} + \left(3H^2 + \frac{k^2}{a^2} + \frac{n^2}{a^2}\right) \mathcal{E}_{(k)} = a^{-3} \sigma_{(k)}, \quad (12b)$$

$$\begin{aligned} \ddot{\mathcal{H}}_{(k)} + 5H\dot{\mathcal{H}}_{(k)} + \left(3H^2 + \frac{k^2}{a^2} + \frac{n^2}{a^2}\right) \mathcal{H}_{(k)} \\ = 2a^{-2} (\dot{\sigma}_{(k)} + 2H\sigma_{(k)}), \end{aligned} \quad (12c)$$

where

$$\begin{aligned} \mathcal{E}_a^{(k)} &= \frac{3}{2} \epsilon_{abc} Q_{(k)}^{bcd} A_d + \frac{1}{2} \epsilon^{bcd} Q_{cda}^{(k)} A_b \\ &+ \epsilon_{abc} Q_{(k)}^{cd} A_{bd} + \epsilon_{abc} Q_{(k)}^{bd} A^c_d, \end{aligned} \quad (13a)$$

and

$$\mathcal{H}_a^{(k)} = Q_{ab}^{(k)} A^b. \quad (13b)$$

It is straight forward to verify that the spatial functions $\mathcal{E}_{(k)}^a$ and $\mathcal{H}_{(k)}^a$ also satisfy the Helmholtz equation.

In order to estimate the dynamical importance of our fields we introduce *expansion normalised variables*

$$\Sigma_{(k)} \equiv \frac{\sigma_{(k)}}{H}, \quad \tilde{\mathcal{E}}_{(k)} \equiv \frac{\mathcal{E}_{(k)}}{H}, \quad \tilde{\mathcal{H}}_{(k)} \equiv \frac{\mathcal{H}_{(k)}}{H}, \quad (14)$$

giving us a set of scale invariant functions (see, e.g. [17]). Introducing the conformal time parameter η (whose defining equation is $\dot{\eta} = a^{-1}$), the scale factor and Hubble parameter for a dust FRW background are given by $a(\eta) = \eta^2$, $H = 2\eta^{-3}$ (see, e.g. [18]).

Substituting these into Eqs. (12) we obtain

$$\Sigma_{(k)}'' + 2\eta^{-1} \Sigma_{(k)}' + (-6\eta^{-2} + k^2) \Sigma_{(k)} = 0, \quad (15a)$$

$$\tilde{\mathcal{E}}_{(k)}'' + 2\eta^{-1} \tilde{\mathcal{E}}_{(k)}' + (k^2 + n^2) \tilde{\mathcal{E}}_{(k)} = \eta^{-2} \Sigma_{(k)}, \quad (15b)$$

and

$$\begin{aligned} \tilde{\mathcal{H}}_{(k)}'' + 2\eta^{-1} \tilde{\mathcal{H}}_{(k)}' + (k^2 + n^2) \tilde{\mathcal{H}}_{(k)} \\ = 2\eta^{-2} (\Sigma_{(k)}' + \eta^{-1} \Sigma_{(k)}). \end{aligned} \quad (15c)$$

Equations (15) can be solved exactly in the long wave length gravitational wave limit (i.e. the term k^2/a^2 is small compared to the other terms in the wave equations), but a numerical investigation gives more transparent results. It turns out to be convenient to introduce the variables $A_{(k,n)} \equiv \tilde{\mathcal{E}}_{(k)} \eta_0^2 / \Sigma(\eta_0)$, $B_{(k,n)} \equiv \tilde{\mathcal{H}}_{(k)} \eta_0^2 / \Sigma(\eta_0)$, where η_0 and $\Sigma(\eta_0)$ are respectively the initial values of the conformal time and the normalised shear, and we have reinstated the index n indicating the scale length of B_0^a . These variables are invariant with respect to changes in η_0 and $\Sigma(\eta_0)$, thus giving us a scale invariant measure of the generated electromagnetic field. Performing the integration for various values of the wave number n for the magnetic test field, we find that the normalised electric field $A_{(k,n)}$ tends to a constant value which depends linearly on the initial value of the shear perturbation, while the normalised magnetic field $B_{(k,n)}$ tends asymptotically to zero (see Figs. 1 & 2 below).

In the long wave length gravitational wave case, the expansion normalised shear is given by $\Sigma_{(k_{\text{long}})} = \frac{3}{4} C_1 \eta^2 - 3C_2 \eta^{-3}$, where C_1 and C_2 are integration constants. For late times, during the matter dominated era (when the equation of state $p = 0$ applies), the second term in $\Sigma_{(k_{\text{long}})}$ can be neglected. In this way we can easily obtain the late time behaviour of the expansion normalised gravitationally induced electric field:

$$\tilde{\mathcal{E}}_{(k_{\text{long}})} = \begin{cases} \frac{1}{12\pi^2} \left(\frac{\lambda_{B_0}}{\lambda_H} \right) \Sigma_{(k_{\text{long}})}, n \neq 0 \\ \frac{1}{6} \Sigma_{(k_{\text{long}})}, n = 0, \end{cases} \quad (16)$$

where $\lambda_H = 1/H$ is the Hubble radius during that epoch. It follows the generated electric field is proportional to the expansion normalised shear. Since the normalised magnetic field asymptotically tends to zero, the above results demonstrate that electric fields produced by this effect could play an important dynamical role in the early universe, possibly causing charge separation. Furthermore, because the asymptotic value of the electric field is proportional to the magnitude of the shear, we can use the CMB anisotropy limits on Σ to give an upper bound on the size of this effect [19]:

$$\tilde{\mathcal{E}}_{(k_{\text{long}})} \lesssim \begin{cases} \left(\frac{\lambda_{B_0}}{\lambda_H} \right) \times 10^{-6}, n \neq 0 \\ 2 \times 10^{-5}, n = 0. \end{cases} \quad (17)$$

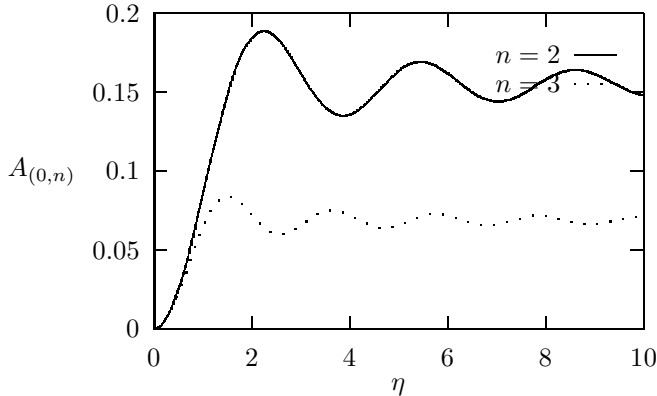


FIG. 1. Expansion normalised gravitationally induced electric field for different values of the magnetic wavenumber n .

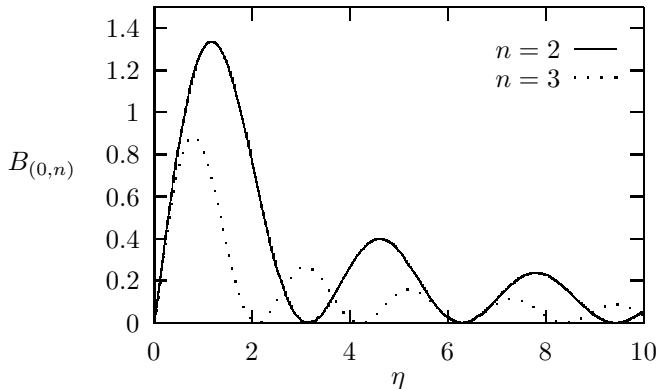


FIG. 2. Expansion normalised gravitationally induced magnetic field for different values of the magnetic wavenumber n .

VI. DISCUSSION

In this paper we derived a set of non-linear wave equations which demonstrate how electromagnetic fields can be generated when gravitational waves couple to an near-homogeneous magnetic test field propagating on a FRW background. In particular we found that for long wavelength gravitational waves, the gravitationally induced fields are proportional to the magnitude of the expansion normalised shear which characterise tensor perturbations. This allows a simple determination of an upper bound on the magnitude of these fields based on the quadrupole anisotropy of the Cosmic Microwave Background.

We note that this paper has not considered the back-reaction of this effect on the gravitational dynamics, which although small may also give rise to interesting results. This issue will be explored in a forthcoming paper [20].

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